INTERPOLATION REVERSE ENGINEERING SYSTEM

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Summary

The article presents the reverse engineering system, which uses the kriging method for interpolation of the spatial irregular net of nodes. It gives the chance for comparison of two clouds of points, which are the results of scanning measurements. The comparison result is the base for modification of machining program for 3-axis milling centre. The final result is the accuracy improvement of the reproducing workpiece.

Keywords: reverse engineering, spatial interpolation

Interpolacyjny system inżynierii rekonstrukcyjnej

Streszczenie

W artykule zaprezentowano system inżynierii odwrotnej, wykorzystujący metodę krigingu do interpolacji przestrzennej nieregularnej siatki węzłów. Stwarza to możliwość porównania dwóch chmur punktów, będących wynikiem pomiarów skanujących. Rezultat porównania jest podstawą do modyfikowania programu sterującego 3-osiowym centrum frezarskim i zwiększenia dokładności odwzorowywanego elementu.

Słowa kluczowe: inżynieria rekonstrukcyjna, interpolacja przestrzenna

1. Introduction

The article shows the reverse engineering system, which uses the geostatistical kriging method [1-5] for interpolation of spatial irregular net of nodes. It gives the chance for comparison of two clouds of points, which are the results of scanning measurements. The comparison result is the base for modification of machining program for 3-axis milling centre. The final result is the accuracy improvement of the reproduced workpiece.

The theory of kriging is described in the first part of the article. Second part shows the schema of the reverse engineering system. The experimental results of system's application are presented in the third part of the paper. The conclusions about the accuracy improvement are described in the last part of the article.

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2. Interpolation of irregular net of nodes

The below problem should be solved:

The set of N points: $P_i(x_i,y_i,f_i)$, $f_i = F(x_i,y_i)$, $i = 1, ..., N$ is given. The function $f(x,y)$ which interpolates the function $F(x,y)$ should be found.

2.1. Ordinary kriging

The ordinary kriging is described by below equation:

$$f(x,y) = \sum_{i=1}^{N} w_i \cdot f_i(x_i, y_i) \tag{1}$$

Using the estimation variance, we can write:

$$\sigma^2 = var\left(f(x,y) - F(x,y)\right) = E\left(\left(f(x,y) - F(x,y)\right)^2\right) \tag{2}$$

where var() is a variance and E() is an expected value.

Next, we can write:

$$\sigma^2 = E\left(\left(f(x,y) - F(x,y)\right)^2\right) =$$

$$= -\gamma(x,y,x,y) - \sum_{i=1}^{N}\sum_{j=1}^{N} w_i \cdot w_j \cdot \gamma(x_i,y_i,x_j,y_j) + 2 \cdot \sum_{i=1}^{N} w_i \cdot \gamma(x_i,y_i,x,y) \tag{3}$$

The minimum of above function with condition:

$$\sum_{i=1}^{N} w_i = 1 \tag{4}$$

should be found.

Using the Lagrange method we can write:

$$\begin{cases} \dfrac{\partial}{\partial w_i}\left(-\gamma(x,y,x,y) - \sum_{i=1}^{N}\sum_{j=1}^{N} w_i \cdot w_j \cdot \gamma(x_i,y_i,x_j,y_j) + \right. \\ \left. \qquad +2 \cdot \sum_{i=1}^{N} w_i \cdot \gamma(x_i,y_i,x,y) + \lambda \cdot \left(\sum_{i=1}^{N} w_i - 1 \right) \right) = 0 \\ \qquad \sum_{i=1}^{N} w_i - 1 = 0 \end{cases} \tag{5}$$

We can substitute:

$$\mu = -\frac{1}{2}\lambda \tag{6}$$

and write the system of linear equations:

$$\begin{cases} \sum_{j=1}^{N} w_j \cdot \gamma(x_i,y_i,x_j,y_j) + \mu = \gamma(x_i,y_i,x,y) \quad i = 1,\dots,N \\ \qquad \sum_{j=1}^{N} w_j = 1 \end{cases} \tag{7}$$

The weights w_j are the solution of the system.

The important part of the ordinary kriging is the function γ, which is named "variogram".

2.2. Variogram

The variogram is a measure of dissimilarity of two values. The variogram for two points: $P_1(x_1,y_1,f_1)$ and $P_2(x_2,y_2,f_2)$ is defined as:

$$\gamma(x_1,y_1,x_2,y_2) = \gamma(P_1,P_2) = \frac{1}{2}\left(f_2 - f_1\right)^2 \tag{8}$$

The variogram is a function of the separation distance between two points, but it is a function of two variables: separation distance h and separation angle θ, which are measured on the plane XY:

$$\left.\begin{aligned} \gamma(P_1,P_2) &= \gamma\left(h(P_1,P_2),\theta(P_1,P_2)\right) \\ h(P_1,P_2) &= \sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2} \\ \theta(P_1,P_2) &= \operatorname{arctg}\frac{y_2-y_1}{x_2-x_1} \end{aligned}\right\} \tag{9}$$

The experimental variogram is approximated by model of variogram.

In 2D-space a representation of the behavior of experimental variogram can be made by drawing a map of iso-variogram lines as a function of separation distance h. Ideally if the iso-variogram lines are circular around the origin, the variogram only depends on the separation distance h and the phenomenon is isotropic (Fig. 1a).

If not, experimental iso-variogram lines can be approximated by concentric ellipses defined along a set of perpendicular main axes of anisotropy (Fig. 1b).

The anisotropic experimental variogram should be transformed into isotropic, because the models of variogram are isotropic – it can be done by rotation and dilatation of anisotropic variogram.

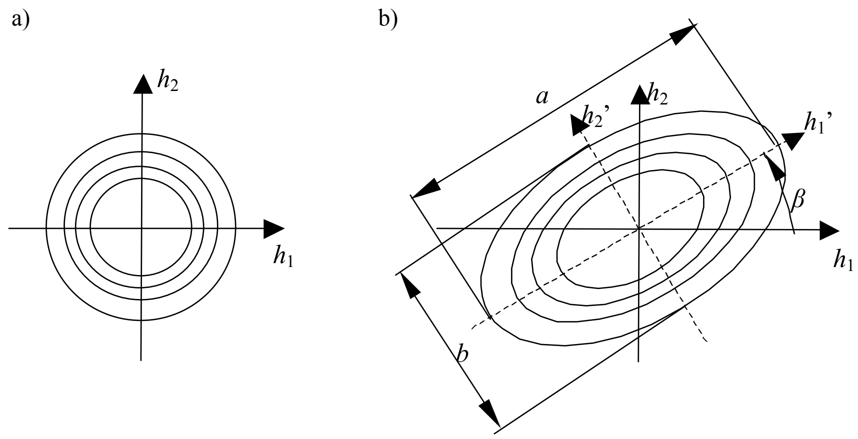


Fig. 1. Behavior of variogram: a) isotropic variogram, b) anisotropic variogram

The separation distance transformed into isotropic model is given by equation [6]:

$$h(P_1,P_2)=\sqrt{\begin{bmatrix}\Delta x & \Delta y\end{bmatrix}\begin{bmatrix}\cos\beta & -\sin\beta\\ \sin\beta & \cos\beta\end{bmatrix}\begin{bmatrix}\left(\frac{1}{a}\right)^2 & 0\\ 0 & \left(\frac{1}{b}\right)^2\end{bmatrix}\begin{bmatrix}\cos\beta & \sin\beta\\ -\sin\beta & \cos\beta\end{bmatrix}\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix}}$$

$$\Delta x = x_2 - x_1 \qquad \Delta y = y_2 - y_1 \tag{10}$$

The bibliography [1, 2, 6, 7] presents some computational models of variogram, for example:

- linear – without anisotropy analysis [6]:

$$\gamma(h) = C \cdot h \tag{11}$$

- exponential [1]:

$$\gamma(h) = C \cdot \left(1 - e^{-h}\right) \tag{12}$$

- rational quadratic [1]:

$$\gamma(h) = C \cdot \left(\frac{h^2}{1+h^2}\right) \tag{13}$$

2.3. Calculation algorithm

The algorithm of kriging interpolation consists of three main part:

1. analysis of variogram for ordinary kriging,
2. calculating the weights,
3. interpolating.

Calculation of experimental variogram should be done for each pair of given points, but the size of calculated values can be extremely large, because the number of pairs is $N(N-1)/2$. For example, 5000 points create 12 497 500 pairs. The above problem is solved by forming the experimental variogram into given classes of separation distance and angle. Moreover, pairs with separation distance larger than arbitrary value are eliminated [6]. The isotropic or anisotropic model of variogram is assumed basing on the calculated classes of separation distance and angle – the function γ will be used in the second part of algorithm.

The second part of algorithm is the equation (7), which can be write as the system of linear equations:

$$\begin{bmatrix} \gamma(P_1,P_1) & \cdots & \gamma(P_1,P_N) & 1 \\ \vdots & \ddots & \vdots & \vdots \\ \gamma(P_N,P_1) & \dots & \gamma(P_N,P_N) & 1 \\ 1 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_N \\ \mu \end{bmatrix} = \begin{bmatrix} \gamma(P_1,P_0) \\ \vdots \\ \gamma(P_N,P_0) \\ 1 \end{bmatrix} \tag{14}$$

where: $P = P(x,y,f)$ and $f = f(x,y)$ is a search function of arbitrary given values (x,y), γ is a function of variogram's model, which was approximated in the first part of algorithm.

Calculation of the weights for all points P_i will be time-consuming – this problem is solved by limitation of number of points P_i for neighbour points in search circle or ellipse.

The result of equation (14) are the weights w_i, which will be used in the next part of algorithm.

The third part of algorithm is equation (1). The result of this equation is an interpolated value of function $f(x,y)$.

2.4. Assessing quality of model for interpolation

Assessing the quality of the model accepted for calculation should be done before interpolation – this procedure is named “Cross validation” [2, 6, 7].

The cross validation is performed in the following steps:

1. the first of point from input data is removed,
2. the interpolation for removed point is calculated using remaining points,
3. the error between true and interpolated values is calculated,
4. the algorithm is repeated for another points,
5. finally, the various statistics of errors are generated and analysed.

2.5. Correction of touch probe radius

The kriging method can be used for correction of radius of touch probe [4]. The direction of correction in point P_0 is defined by the versor $\vec{w}$, which is normal to the surface:

$$\vec{n} = [f_x, f_y, -1], \quad f_x = \frac{\partial f}{\partial x}, \quad f_y = \frac{\partial f}{\partial y}, \quad \vec{w} = \frac{\vec{n}}{|\vec{n}|} \tag{15}$$

The derivates f_x and f_y can be obtained from equations (see Fig. 2):

$$\left.\begin{aligned} f_x &= \frac{f(x_0+\Delta x, y_0) - f(x_0-\Delta x, y_0)}{2\cdot\Delta x} \\ f_y &= \frac{f(x_0, y_0+\Delta y) - f(x_0, y_0-\Delta y)}{2\cdot\Delta y} \end{aligned}\right\} \quad (16)$$

The points P_1, P_2, P_3 and P_4 can be interpolated by kriging method.

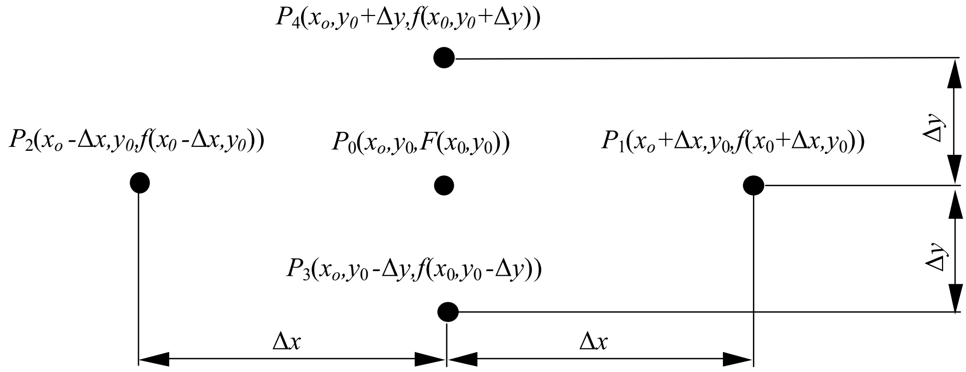


Fig. 2. Calculation of derivates

3. Reverse engineering system

The comparison of two surfaces, which were measured on the coordinate measuring machine, is the serious problem in the reverse engineering system. These problems are connected with the fact, that the both surfaces are described by cloud of points. Next typical problem of reverse engineering is the estimation of accuracy of model's surface, which was machined basing on the scanned surface. The simple way of estimation is a comparison of *Z*-coordinate in the same *X* and *Y* coordinates of adequate points on the surfaces – namely, the same grid *X,Y*-coordinates should be constructed for both surfaces. The kriging method gives the tool for generating grids with errors, which can be skipped.

The schema of proposed reverse engineering system is shown in Fig. 3.

The unknown surface is digitized (1 and 2 in Figure 3) using the coordinate measuring machine. The virtual model of scanned surface is constructed in the computer aided design (CAD) system. The machining program for the milling centre (4) is prepared in the computer aided manufacturing (CAM) system (3). The machined workpiece is repeat digitized (5) and the assessing of accuracy is carried out. Next, the correction of machined program can be done.

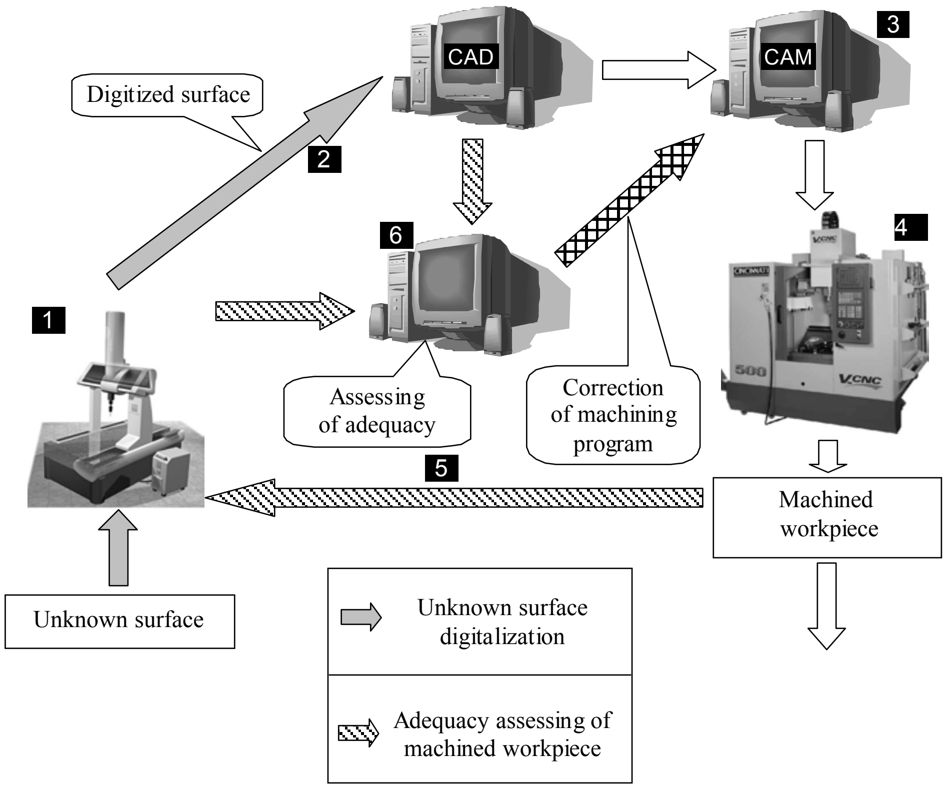


Fig. 3. Schema of reverse engineering system

4. Application of reverse engineering system

The constructed system has been used for the assessing of accuracy in reverse engineering. The algorithm of described assessing is shown in Fig. 4.

The experiment of accuracy assessing has been done using the model, which is shown in Fig. 5.

The surface of element has been defined as Coons surface with curve border on the cube with dimensions 60 mm x 60 mm x 8 mm. On account of machining, the cube has been put into the cylinder and placed on the cuboid base. The element has been scanned using the PMM12106 coordinate measuring machine with 2 mm radius touch probe. Distance between programmed trajectory has been 1 mm, and the 2 points on 1 mm has been acquired.

The scanning results (coordinates of probe centre) have been interpolated by kriging method using the grid with dimension 1 mm x 1 mm on XY-plane.

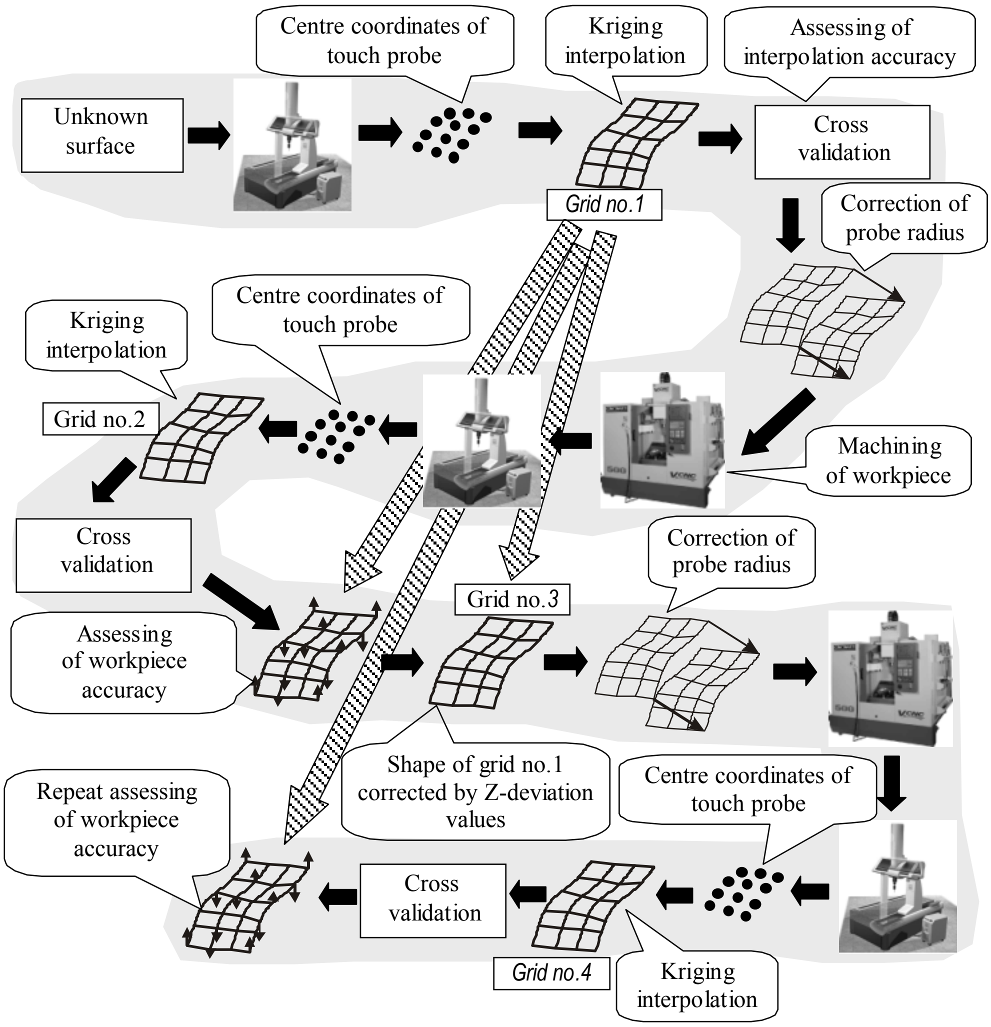


Fig. 4. Algorithm of adequacy assessing

The interpolation accuracy of kriging with various models of variogram has been calculated before the main part of reverse engineering algorithm, using the cross validation. The best results of interpolation accuracy has been obtained by kriging with rational quadratic variogram (mean error 0.0001 mm, standard deviation of error 0.0042 mm). This variogram model has been accepted for future interpolation in the algorithm.

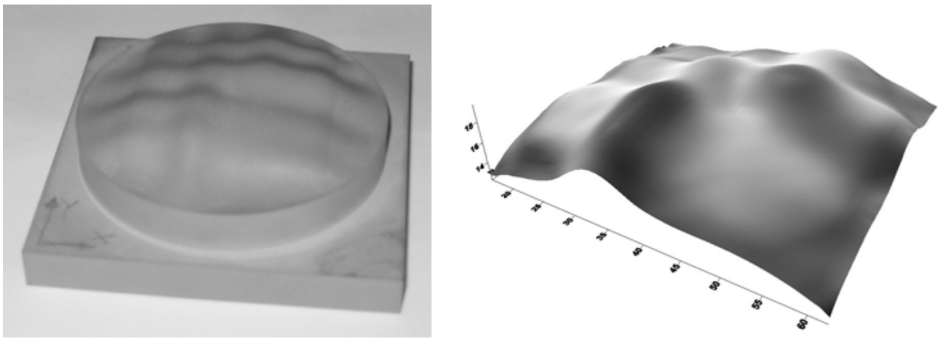


Fig. 5. The surface of reproduced element and its virtual model

The kriging with rational quadratic model of variogram has been used for interpolation of scanning results. The interpolated grid 1 mm x 1 mm describes the surface of probe centre – the correction of probe radius is required. The correction has been done using the interpolation method, which has been described in earlier part of the article.

The corrected points have been used for constructing the surface of virtual model of the element and the machined program for milling centre has been prepared. The copy of original element has been machined in 3-axis miling centre Arrow 500 (material of workpiece: poliuretan resin Prolab 60; tool: sinking miling cutter D8; machining cycle: sweeping; scallop height: 0.001 mm; trajectory tolerance: 0.001 mm).

The machined element has been repeat scanned. The measured points of probe centre have been interpolated using kriging with rational quadratic model of variogram (interpolation errors: mean error 0.0001 mm, standard deviation 0.0038 mm).

Because the both interpolated surfaces have the same grid, we can compare the surface, calculating the differences between the values of Z-coordinates of points with the same X,Y-coordinates – it is the error of reproducing:

$$\Delta z = z_{x,y}^{(1)} - z_{x,y}^{(2)} \tag{17}$$

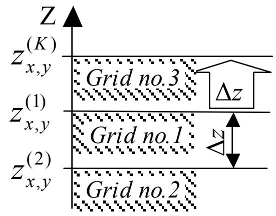
where the upper indexes (1) and (2) describe the model and the reproduced surfaces.

The mean reproducing error was 0.015 mm with standard deviation 0.025 mm.

On the actual stage of algorithm we have the information about the distribution of errors, and we can try to reduce these errors. It has been done using the formula:

$$z_{x,y}^{(K)} = z_{x,y}^{(1)} + \Delta z \tag{18}$$

where: $z_{x,y}^{(K)}$ – corrected value of Z-coordinate, $z_{x,y}^{(1)}$ – Z-coordinate value original model, Δ – value of reproducing error in point (X,Y).



The presented method "deforms" the original model surface using the calculated values of the reproducing errors – it means, that the corrected shape of surface takes into consideration the reproducing errors, but without analysis of reason of these errors.

The corrected surface (grid) has been transformed like previous surface and the next copy of element has been machined, which has been scanned and interpolated (interpolation errors: mean error 0.001 mm, standard deviation 0.0046 mm).

The assessing of reproducing accuracy has been done – mean error 0.009 mm (previous value 0.015 mm), standard deviation 0.019 mm (previous value 0.025 mm). The previous and actual distributions of errors are shown in Fig. 6.

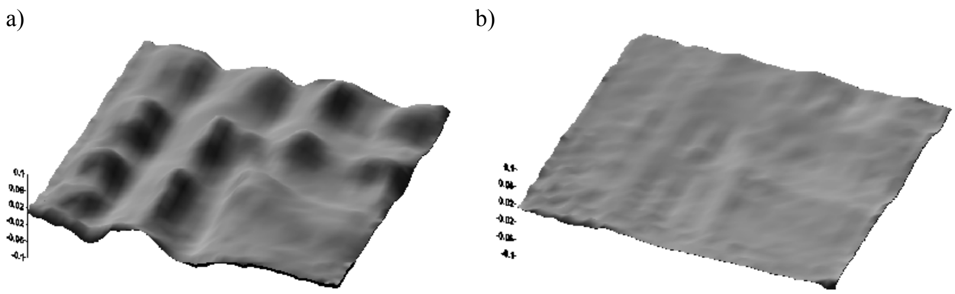


Fig. 6. Distributions of errors: before (a) and after (c) correction

5. Conclusions

The presented reverse engineering system uses the geostatistical kriging interpolation method, which gives the tool for interpolating the cloud of points, which has been acquired by coordinate measuring machine.

The calculation of the Z-value for arbitrary given X,Y-coordinates enables the comparison of two sets of points by evaluation of Z-coordinates only.

The kriging interpolation can be used for correction of touch probe radius, too.

The method proposed by author can be used for assessing and increasing the accuracy of reproducing elements in the reverse engineering system. This method "deforms" the surface of original model using the calculated values of the reproducing errors – it means, that the corrected shape of surface takes into consideration the reproducing errors, but without analysis of reason of these errors.

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